Final Year B.Sc. Degree Examination, March 2009
Part – III : Group – I : MATHEMATICS
Paper IV – Differential Equations, Numerical Analysis and Vectors
(Perior to 2006 Admission)

Time : 3 Hours

Max. Marks : 65

Instruction : Maximum of 13 marks can be earned from each Unit.

UNIT – I

1. Solve \( \frac{dy}{dx} = \frac{y - x + 1}{y - x + 5} \).

2. Show that the equation \( (x^2 - 4xy - 2y^2) \, dx + (y^2 - 4xy - 2x^2) \, dy = 0 \) is exact and hence solve it.

3. Find the orthogonal trajectories of the circles \( x^2 + (y - c)^2 = c^2 \).

4. Solve : \( (D^2 - 1) \, y = 2x^2 \).

5. Solve : \( (D^2 - 2D + 2) \, y = e^x \cos 2 \, x \).

UNIT – II

6. Solve : \( x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x \).

7. Solve the system
\[
\begin{align*}
\frac{dx}{dt} &= x + y \\
\frac{dy}{dt} &= 4x + y.
\end{align*}
\]

P.T.O.
8. Find the Laplace transforms of
   \( e^{-t} \cos 2t \) and \( 4e^{5t} + 6t^3 - 3 \sin 4t \).

9. Solve the equation \( y'(t) + y(t) = t, \ y(0) = 1, \ y'(0) = -2 \), using Laplace transforms.

UNIT – III

10. Prove that \( \Delta^n \sin (ax + h) = (2 \sin \frac{ah}{2})^n \sin \left[ ax + h + \frac{n}{2} (ah + \pi) \right] \).

11. Prove that i) \( 1 + \Delta = E \)
    
    ii) \( 1 - \nabla = E^{-1} \).

12. The following data gives the melting point of an alloy of lead and zinc, where \( t \) is the temperature and \( p \) is the percentage of lead in the alloy.

<table>
<thead>
<tr>
<th>( p )</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>226</td>
<td>250</td>
<td>276</td>
<td>304</td>
</tr>
</tbody>
</table>

Applying Newton’s interpolation formula, find the melting point of the alloy containing 84 percent of lead.

13. Apply Lagrange’s formula to find \( f(5) \) given that \( f(1) = 2, f(2) = 4, f(3) = 8, f(4) = 16, f(7) = 128 \).

UNIT – IV

14. Prove that \( (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c}) \).

15. If \( |\vec{r}| = r \), where \( \vec{r} = xi + yj + zk \), prove that \( \nabla \vec{r} = \frac{\vec{r}}{r} \).

16. Find the directional derivative of the function \( 2xy + z^2 \) in the direction of the vector \( \vec{i} + 2\vec{j} + 2\vec{k} \) at the point \((1, -1, 3)\).
17. Show that \( \mathbf{F} = (2xy + z^3)i + x^2j + 3xz^2k \) is a conservative force field. Find the scalar potential.

18. If \( \mathbf{F} \) is any vector point function, prove that \( \text{div} (\text{Curl} \, \mathbf{F}) = 0 \).

UNIT - V

19. Evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F} = x^2i + y^2j \) and \( C \) is the arc of the parabola \( y = x^2 \) in the \( xy \)-plane from \((0, 0)\) to \((1, 1)\).

20. Evaluate \( \iint_S \mathbf{F} \cdot \mathbf{n} \, ds \), where \( \mathbf{F} = 6z\mathbf{i} - 4\mathbf{j} + y\mathbf{k} \), where \( S \) is the portion of the plane \( 2x + 3y + 6z = 12 \) in the first octant.

21. State Green’s Theorem.

22. Apply Stoke’s theorem to evaluate \( \int_C (y \, dx + z \, dy + x \, dz) \) where \( C \) is the curve of intersection of \( x^2 + y^2 + z^2 = a^2 \) and \( x + y = a \).