Final Year B.Sc. Degree Examination, March 2009
Part – III : Group – I : MATHEMATICS
Paper – VI : Complex Variables
(Prior to 2006 Admission)

Time : 3 Hours  Max. Marks : 65

Instruction : The maximum marks for each Unit is 13.

UNIT – I

1. Prove that $|z_1 - z_2| \leq |z_1 + z_2|$. 4

2. Obtain the complex equation of a straight line through two given points $Z_1$ and $Z_2$. 4

3. State and prove Cauchy-Riemann Equations. 6

4. If a function $f(z)$ is analytic, show that it is independent of $\bar{Z}$. 3

5. Given $u = y^3 - 3x^2y$, find $f(z) = u + iv$, such that $f(z)$ is analytic. 3

UNIT – II

6. Write the inverse of the bilinear transformation $w = \frac{az + b}{cz + d}$, where $ad - bc \neq 0$. 2

7. Find the bilinear transformation which maps $Z = 1, i, -1$ into the points $w = 0, 1, \infty$. 4

8. Discuss the transformation $w = e^z$. 6

P.T.O.
9. Find the region in the w-plane corresponding to the region \(|z| \leq 1\) under the transformation \(w = \frac{1+z}{1-z}\).

10. Show that under \(w = \frac{1}{z}\) circles straight lines in the z-plane is mapped into circles or straight lines in the w-plane.

UNIT III

11. Evaluate \(\int_C \frac{1}{z^2(z-1)} dz\) such that \(|z| = \frac{3}{4}\).

12. From the integral \(\int_C \frac{1}{z+2} dz\) where C is \(|Z| = 1\). Show that \(\int_0^\pi \frac{1+2\cos \theta}{5+4\cos \theta} d\theta = 0\).

13. State and derive Cauchy’s Integral formula.

14. State and prove Cauchy’s inequality for \(f^n(z_0)\).

15. State fundamental theorem of algebra.

UNIT IV

16. State and Prove Laurent’s theorem.

17. Find the Taylor series expansion of \(\frac{z^2-1}{(z+2)(z+3)}\) in \(|z| < 2\).
18. Show that the Laurent’s expansion of \( \frac{1}{2-z} + \frac{1}{z-1} \) in \( 1 < |z| < 2 \) is

\[
\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^n + \sum_{n=1}^{\infty} \frac{1}{z^n} .
\]

19. Find the radius of convergence of

\[
i) \sum \frac{1}{n^2} z^n \quad \text{ii) } \sum 2^{\sqrt{n}} z^n .
\]

UNIT – V

20. Define Pole of order m.

21. State and prove Rouche’s theorem.

22. Find the zeros and nature of singularities of

\[
i) \frac{\sin z}{z^4} \quad \text{ii) } \frac{1}{\sin \left( \frac{\pi}{z} \right)} .
\]

23. Evaluate \( \int_{0}^{\infty} \frac{x \sin x}{1+x^2} \, dx \).

24. Evaluate \( \int_{0}^{2\pi} \frac{\cos 2\theta}{5+4 \sin \theta} \, d\theta \).