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Generalized Fuzzy Ideal Closed Sets on Fuzzy Topological Spaces in Sostak Sense

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Abstract

Recently, El-Naschie has shown that the notion of fuzzy topology may be relevant to quantum paretical physics in connection with string theory and E-infinity space time theory. In this paper, we define concept r-generalized fuzzy ideal closed sets with respect to an fuzzy ideal topological space in Sostak sense.We investigate some properties of them, we investigate the relationships between r-generalized fuzzy ideal closed sets with respect to an ideal and r-fuzzy separated

Keywords: r-generalized fuzzy closed sets, r-generalized fuzzy closed sets with respect to an fuzzy ideal topological space in Sostak sense, r-fuzzy separated

1. Introduction

Sostak [1], introduce the fundamental concept of fuzzy topological structure as an extension of both crisp topology and Chang's fuzzy topology [2], in the sense that not only the object were fuzzified, but also the axiomatics. Chattopdhyay et al. [3] and El Naschie [4] have redefined the similar concept. In [5], the author gave a similar definition namely "Smooth fuzzy topology". We must point out that; the concept of fuzzy topological spaces, has been a significant concept in string theory and *E-infinity* theory pertaining to quantum particular physics ever since El-Naschie [6–14]. After that several authors [15–17] have introduced the smooth definition and studied smooth fuzzy ideai topological spaces being unaware of Sostak works.

Throughout this paper, let X be a nonempty set $I = [0, 1]$ and $I_0 = (0, 1]$. For $\alpha \in I$, $\overline{\alpha}(x) = \alpha$ for all $x \in X$. The family of all fuzzy sets on X denoted by I^X. For two fuzzy sets we write $\lambda q \mu$ to mean that λ is quasi-coincident (q-coincident, for short) with μ, i.e, there exists at least one point $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Negation of such a statement is denoted as $\lambda \overline{q} \mu$.

Definition 1.1 ([1]). A mapping $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:

(O1) $\tau(\overline{0}) = \tau(\overline{1}) = \overline{1}$.

(O2) $\tau(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \tau(\mu_i)$, for any $\{\mu_i\}_{i\in\Gamma} \in I^X$.

(O3) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1, \mu_2 \in I^X$.

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The pair (X, τ) is called a fuzzy topological space (for short, fts).

Definition 1.2 ([18]). Let (X, τ) be a fts, $\lambda, \mu \in I^X$ and $r \in I_0$.

1) A fuzzy set λ is called r-generalized fuzzy closed (for short, r-gfc) if $C_\tau(\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$ and $\tau(\mu) \geq r$.

2) A fuzzy set λ is called r-generalized fuzzy open (for short, r-gfo) if $I_\tau(\lambda, r) \geq \mu$ whenever $\lambda \geq \mu$ and $\tau(\overline{1}-\mu) \geq r$.

Definition 1.3 ([1, 5, 15, 16]). A mapping $\mathbf{I} : I^X \to I$ is called fuzzy ideal on X iff:

1) $\mathbf{I}(\underline{0}) = 1, \mathbf{I}(\underline{1}) = 0$.

2) If $\lambda \leq \mu$, then $\mathbf{I}(\lambda) \geq \mathbf{I}(\mu)$, for each $\lambda, \mu \in I^X$.

3) For each $\lambda, \mu \in I^X$, $\mathbf{I}(\lambda \vee \mu) \geq \mathbf{I}(\lambda) \wedge \mathbf{I}(\mu)$ [finite additivity].

Lemma 1.4. Let $(X, \tau, \mathcal{I})$ be a fits. The simplest fuzzy ideal on X are $\mathcal{I}^0, \mathcal{I}^1 : I^X \to I$ where

$$\mathcal{I}^1(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{0}, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathcal{I}^0(\lambda) = \begin{cases} 0, & \text{if } \lambda = \underline{1}, \\ 1, & \text{otherwise.} \end{cases}$$

If we take $\mathcal{I} = \mathcal{I}^0$, for each $\mathcal{A} \in I^X$ we have $\mathcal{A}^*_r = C_\tau(\mathcal{A}, r)$.

If we take $\mathcal{I} = \mathcal{I}^1$, for each $\mathcal{A} \in \Theta'$ we have $\mathcal{A}^*_r = \underline{0}$, where, $\underline{1} \notin \Theta'$ be a subset of I^X.

Definition 1.5 ([19]). Let $(X, \tau, \mathbf{I})$ be a fuzzy ideal topological space. Let $\mu, \lambda \in I^X$, the r-fuzzy open local function μ^*_r of μ is the union of all fuzzy points x_t such that if $\rho \in Q(x_t, r)$ and $\mathbf{I}(\lambda) \geq r$ then there is at least one $y \in X$ for which $\rho(y) + \mu(y) - 1 > \lambda(y)$.

Theorem 1.6 ([4]). Let (X, τ) be a fts. Then for each $r \in I_0$, $\lambda \in I^X$ we define an operator $C_\tau : I^X \times I_0 \to I^X$ as follows:

$$C_\tau(\lambda, r) = \bigwedge\{\mu \in I^X : \lambda \leq \mu,\ \tau(\overline{1} - \mu) \geq r\}.$$

For $\lambda, \mu \in I^X$ and $r, s \in I_0$, the operator C_τ satisfies the following conditions:

1) $C_\tau(\overline{0}, r) = \overline{0}$.

2) $\lambda \leq C_\tau(\lambda, r)$.

3) $C_\tau(\lambda, r) \vee C_\tau(\mu, r) = C_\tau(\lambda \vee \mu, r)$.

4) $C_\tau(\lambda, r) \leq C_\tau(\lambda, s)$ if $r \leq s$.

5) $C_\tau(C_\tau(\lambda, r), r) = C_\tau(\lambda, r)$.

Theorem 1.7 ([20]). Let (X, τ) be a fts. Then for each $r \in I_0$, $\lambda \in I^X$ we define an operator $I_\tau : I^X \times I_0 \to I^X$ as follows:

$$I_\tau(\lambda, r) = \bigvee\{\mu \in I^X : \lambda \geq \mu,\ \tau(\mu) \geq r\}.$$

For $\lambda, \mu \in I^X$ and $r, s \in I_0$, the operator I_τ satisfies the following conditions:

1) $I_\tau(\overline{1} - \lambda, r) = \overline{1} - C_\tau(\lambda, r)$ and $C_\tau(\overline{1} - \lambda, r) = \overline{1} - I_\tau(\lambda, r)$.

2) $I_\tau(\overline{1}, r) = \overline{1}$.

3) $\lambda \geq I_\tau(\lambda, r)$.

4) $I_\tau(\lambda, r) \wedge I_\tau(\mu, r) = I_\tau(\lambda \wedge \mu, r)$.

5) $I_\tau(\lambda, r) \leq I_\tau(\lambda, s)$ if $r \geq s$.

6) $I_\tau(I_\tau(\lambda, r), r) = I_\tau(\lambda, r)$.

2. r-generalized Fuzzy Closed Sets with Respect to an Ideal

Definition 2.1. Let $(X, \tau, \mathbf{I})$ be fuzzy ideal topological space, $\mu \in I^X$ and $r \in I_0$. A fuzzy set μ is called r-generalized fuzzy closed with respect to an ideal (briefly, r-gfIc) if $\mathbf{I}(C_\tau(\mu, r) \setminus \lambda) \geq r$, whenever $\mu \leq \lambda$ and $\tau(\lambda) \geq r$.

Lemma 2.2. Every r-gfc set is r-gfIc.

Proof. Let $\mu \leq \lambda$ and $\tau(\lambda) \geq r$. Since μ is r-gfc set, then $C_\tau(\mu, r) \leq \lambda$, this implies that $C_\tau(\mu, r)\overline{q}\underline{1} - \lambda$, implies $C_\tau(\mu, r)(x) + (\underline{1} - \lambda)(x) \leq 1$, then $C_\tau(\mu, r)(x) - \lambda(x) \leq 0$. Thus, $\mathbf{I}(C_\tau(\mu, r) \setminus \lambda) \geq r$. □

Example 2.3. The converse Lemma 2.2 is not true. Let $X = \{a, b, c\}$ be a set and $\alpha, \beta, \gamma \in I^X$ are defined as follows:

$$\alpha(a) = 0.2,\ \alpha(b) = 0.4;\ \alpha(c) = 0.7$$
$$\beta(a) = 0.7,\ \beta(b) = 0.6;\ \beta(c) = 0.8$$
$$\gamma(a) = 0.6;\ \gamma(b) = 0.4,\ \gamma(c) = 0.7.$$

We define fuzzy topology and fuzzy ideal $\tau, \mathbf{I} : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \nu = \underline{1}, \underline{0}, \\ \frac{1}{2}, & \text{if } \nu = \alpha, \\ \frac{1}{2}, & \text{if } \nu = \beta, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathbb{I}(\lambda) = \begin{cases} 1, & \text{if } \nu = \underline{0}, \\ \frac{1}{2}, & \text{if } \nu = \underline{0.3}, \\ \frac{1}{2}, & \text{if } \underline{0} < \nu < \underline{0}.3, \\ 0, & \text{otherwise,} \end{cases}$$

For $r = \frac{1}{3}$, $\underline{1} - \gamma$ is r-gfIc set, where

$$\underline{1} - \gamma \leq \beta, \ \tau(\beta) \geq \frac{1}{3},$$
$$C_\tau(\underline{1} - \gamma, \frac{1}{3}) = \underline{1} - \alpha \setminus \beta = a_{0.3}.$$

Therefore, $\mathbf{I}((C_\tau(\underline{1} - \gamma, \frac{1}{3}) \setminus \alpha), \frac{1}{3}) \geq \frac{1}{3}$.

But $\underline{1} - \gamma$ is not r-gfc set because

$$\underline{1} - \gamma \leq \beta, \ \tau(\beta) \geq \frac{1}{3}, \ (C_\tau(\underline{1} - \gamma, \ \frac{1}{3}) = \underline{1} - \alpha) \geq \beta.$$

Theorem 2.4. Let $(X, \tau, \mathbf{I})$ be an fuzzy ideal topological space, $\mu, \ \lambda \in I^X$ and $r \in I_0$. If μ and λ are r-gfIc sets, then $\mu \vee \lambda$ is r-gfIc.

Proof. Suppose μ and λ are r-gfIc sets. If $\mu \vee \lambda \leq \rho$ and $\tau(\rho) \geq r$, then $\mu \leq \rho$ and $\lambda \leq \rho$. By assumption, $\mathbf{I}(C_\tau(\mu, r) \setminus \rho) \geq r$ and $\mathbf{I}(C_\tau(\lambda, r) \setminus \rho) \geq r$ and hence

$$\mathbf{I}(C_\tau(\mu \vee \lambda, r) \setminus \rho = C_\tau(\mu, r) \setminus \rho \vee C_\tau(\lambda, r) \setminus \rho) \geq r.$$

Therefore, $\mu \vee \lambda$ is r-gfIc. □

Remark 2.5. The intersection of two r-gfIc sets need not be an r-gfIc set as shown by the following example.

Example 2.6. Let $X = \{a, b, c\}$ be a set and $\alpha, \beta, \gamma \in I^X$ are defined as follows:

$$\alpha(a) = 0.8, \ \alpha(b) = 0.4; \ \alpha(c) = 0.7$$
$$\beta(a) = 0.6, \ \beta(b) = 0.5; \ \beta(c) = 0.8$$
$$\gamma(a) = 0.6; \ \gamma(b) = 0.4, \ \gamma(c) = 0.7.$$

We define fuzzy topology and fuzzy ideal $\tau, \mathbf{I} : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \nu = \underline{1}, \underline{0}, \\ \frac{1}{2}, & \text{if } \nu = \gamma, \\ 0, & \text{otherwise,} \end{cases}$$

$$\mathbb{I}(\lambda) = \begin{cases} 1, & \text{if } \nu = \underline{0}, \\ \frac{1}{2}, & \text{if } \nu = \underline{0.3}, \\ \frac{1}{2}, & \text{if } \underline{0} < \nu < \underline{0}.3, \\ 0, & \text{otherwise.} \end{cases}$$

For $r = \frac{1}{3}$, β and γ is r-gfIc set. But $\beta \wedge \gamma = \gamma$ is not r-gfIc set because $\gamma \leq \gamma, \quad \tau(\gamma) \geq r, \ C_\tau(\gamma, \frac{1}{3}) = \underline{1} \setminus \gamma = \underline{1}$. Therefore $\mathbf{I}((C_\tau(\gamma, \frac{1}{3}) \setminus \gamma), \frac{1}{3}) < \frac{1}{3}$.

Theorem 2.7. Let $(X, \tau, \mathbf{I})$ be an fuzzy ideal topological space, $\mu, \ \lambda \in I^X$ and $r \in I_0$. If μ is r-gfIc set and $\mu \leq \lambda \leq C_\tau(\mu, r)$, then λ are r-gfIc.

Proof. Let μ is r-gfIc set and $\mu \leq \lambda \leq C_\tau(\mu, r)$. Suppose $\lambda \leq \rho$ and $\tau(\rho) \geq r$. Then $\mu \leq \rho$. Since μ is r-gfIc, we have $\mathbf{I}(C_\tau(\mu, r) \setminus \rho) \geq r$. Now $\lambda \leq C_\tau(\mu, r)$ implies that

$$C_\tau(\lambda, r) \setminus \rho \leq C_\tau(\mu, r) \setminus \rho,$$

and hence, $\mathbf{I}(C_\tau(\lambda, r) \setminus \rho) \geq r$. Therefore, λ is r-gfIc set. □

Definition 2.8. Let $(X, \tau, \mathbf{I})$ be fuzzy ideal topological space, $\mu \in I^X$ and $r \in I_0$. A fuzzy set μ is called r-fuzzy generalized open with respect to an ideal $\mathbf{I}$ (briefly, r-gfIo) if $\underline{1} - \mu$ is r-gfIc set.

Theorem 2.9. Let $(X, \tau, \mathbf{I})$ be an fuzzy ideal topological space, $\mu, \ \lambda, \ \rho \in I^X$ and $r \in I_0$. If μ is r-gfIo sets if and only if $\lambda \setminus \rho \leq I_\tau(\mu, r)$ for some $\mathbf{I}(\rho) \geq r$, whenever $\lambda \leq \mu$ and $\tau(\underline{1} - \lambda) \geq r$.

Proof. Suppose that μ is r-gfIo sets. Suppose $\lambda \leq \mu$ and $\tau(\underline{1} - \lambda) \geq r$. We have $\underline{1} - \lambda \geq \underline{1} - \mu$. By assumption,

$$C_\tau(\underline{1} - \mu, r) \leq \underline{1} - \lambda \vee \rho.$$

For some $\mathbf{I}(\rho) \geq r$. This implies

$$\underline{1} - ((\underline{1} - \lambda) \vee \rho) \leq \underline{1} - C_\tau(\underline{1} - \mu),$$

and hence, $\lambda \setminus \rho \leq I_\tau(\mu, r)$.

Conversely, assume that $\lambda \leq \mu$ and $\tau(\underline{1} - \lambda) \geq r$ imply $\lambda \setminus \rho \leq I_\tau(\mu, r)$ for some $\mathbf{I}(\rho) \geq r$. Consider $\tau(\omega) \geq r$ such that $\underline{1} - \mu \leq \omega$. Then $\underline{1} - \omega \leq \mu$. By assumption,

$$\underline{1} - \omega \setminus \rho \leq I_\tau(\mu, r) = \underline{1} - C_\tau(\underline{1} - \mu, r)$$

For some $\mathbf{I}(\rho) \geq r$. This gives that

$$\underline{1} - (\omega \vee \rho) \leq \underline{1} - C_\tau(\underline{1} - \mu, r).$$

Therefore, $C_\tau(\underline{1} - \mu, r) \leq \omega \vee \rho$, for some $\mathbf{I}(\rho) \geq r$. This show that $\mathbf{I}(C_\tau(\underline{1} - \mu, r) \setminus \omega) \geq r$. Hence, $\underline{1} - \mu$ is r-gfIc set. □

Recall that the sets μ and λ are fuzzy separated if $C_\tau(\mu, r)\overline{q}\lambda$ and $\mu\overline{q}C_\tau(\lambda, r)$.

Theorem 2.10. Let $(X, \tau, \mathbf{I})$ be an fuzzy ideal topological space, $\mu,\ \lambda, \in I^X$ and $r \in I_0$. If μ and λ are fuzzy separated and r-gfIo sets, then $\mu \vee \lambda$ is r-gfIo.

Proof. Suppose μ and λ are fuzzy separated and r-gfIo sets and $\rho \leq \mu \vee \lambda$, and $\tau(\underline{1} - \rho) \geq r$. Then $\rho \wedge C_\tau(\mu, r) \leq \mu$ and $\rho \wedge C_\tau(\lambda, r) \leq \lambda$. By assumption,

$$\rho \wedge C_\tau(\mu, r) \setminus \nu_1 \leq I_\tau(\mu, r),$$

$$\rho \wedge C_\tau(\lambda, r) \setminus \nu_2 \leq I_\tau(\lambda, r),$$

for some $\mathbf{I}((\nu_1, \nu_2), r) \geq r$. This means

$$\mathbf{I}(\rho \wedge C_\tau(\mu, r) \setminus I_\tau(\mu, r), r) \geq r,$$

and

$$\mathbf{I}(\rho \wedge C_\tau(\lambda, r) \setminus I_\tau(\lambda, r), r) \geq r.$$

Thus,

$$\mathbf{I}((\rho \wedge C_\tau(\mu, r) \setminus I_\tau(\mu, r)) \vee (\rho \wedge C_\tau(\lambda, r)) \setminus I_\tau(\lambda, r), r) \geq r.$$

Therefore,

$$\mathbf{I}(\rho \wedge (C_\tau(\mu, r) \vee C_\tau(\lambda, r)) \setminus (I_\tau(\mu, r) \vee I_\tau(\lambda, r)), r) \geq r.$$

But

$$\rho = \rho \wedge (\mu \vee \lambda) \leq \rho \wedge (C_\tau(\mu \vee \lambda, r)),$$

and we have

$$\mathbf{I}(\rho \setminus I_\tau(\mu \vee \lambda, r) \leq (\rho \wedge C_\tau(\mu \vee \lambda, r)) \setminus I_\tau(\mu \vee \lambda, r) \leq (\rho \wedge C_\tau(\mu \vee \lambda, r)) \setminus (I_\tau(\mu, r) \vee I_\tau(\lambda, r))) \geq r.$$

Hence, $\rho \setminus \nu \leq Int_\tau(\mu \vee \lambda, r)$ for some $\mathbf{I}(\nu) \geq r$. This proves that $\mu \vee \lambda$ is r-gfIo. □

Corollary 2.11. Let $(X, \tau, \mathbf{I})$ be an fuzzy ideal topological space, $\mu,\ \lambda, \in I^X$ and $r \in I_0$. If μ and λ are r-gfIo sets, $\underline{1} - \mu$ and $\underline{1} - \lambda$ are fuzzy separated. Then $\mu \wedge \lambda$ is r-gfIc.

Proof. Obvious. □

Corollary 2.12. Let $(X, \tau, \mathbf{I})$ be an fuzzy ideal topological space, $\mu,\ \lambda, \in I^X$ and $r \in I_0$. If μ and λ are r-gfIo sets, then $\mu \wedge \lambda$ is r-gfIo.

Proof. Obvious. □

Theorem 2.13. Let $(X, \tau, \mathbf{I})$ be an fuzzy ideal topological space, $\mu,\ \lambda, \in I^X$ and $r \in I_0$. If and $\mu \leq \lambda$, and μ r-gfIo relative to λ and λ is r-gfIo relative to X, then μ r-gfIo relative to X.

Proof. Suppose that $\mu \leq \lambda$, μ is r-gfIo relative to λ and λ is r-gfIo relative to X. Let $\rho \leq \mu$ and $\tau(\underline{1} - \rho) \geq r$. Since μ is r-gfIo relative to λ. By Theorem 2.9, $\rho \setminus \nu_1 \leq I_\lambda(\mu, r)$ for some $\mathbf{I}(\nu_1) \geq r$. This implies that there exists $\tau(\omega_1) \geq r$ such that

$$\rho \setminus \nu_1 \leq \omega_1 \wedge \lambda \leq \mu,$$

for some $\mathbf{I}(\nu_1) \geq r$. Let $\rho \leq \lambda$ and $\tau(\underline{1} - \rho) \geq r$. Since λ is r-gfIo, we have

$$\rho \setminus \nu_2 \leq Int_\tau(\lambda, r)$$

for some $\mathbf{I}(\nu_2) \geq r$. This implies that there exists $\tau(\omega_2) \geq r$ such that

$$\rho \setminus \nu_2 \leq \omega_2 \leq \lambda,$$

for some $\mathbf{I}(\nu_2) \geq r$. Now

$$\rho \setminus (\nu_1 \vee \nu_2) = (\rho \setminus \nu_1) \wedge (\rho \setminus \nu_2) \leq \omega_1 \wedge \omega_2 \leq \omega_1 \wedge \lambda \leq \mu.$$

This implies that $\rho \setminus (\nu_1 \vee \nu_2) \leq I_\lambda(\mu, r)$ for some $\mathbf{I}(\nu_1 \vee \nu_2) \geq r$. Thus, μ r-gfIo relative to X. □

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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References

[1] A. P. Sostak, "On a fuzzy topological structure," in *Proceedings of the 13th Winter School on Abstract Analysis (Circolo Matematico di Palermo)*, Palermo, Italy, pp. 89-103, 1985.

[2] C. L. Chang, "Fuzzy topological spaces," *Journal of Mathematical Analysis and Applications*, vol. 24, no. 1, pp. 182-190, 1968.

[3] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, "Gradation of openness: fuzzy topology," *Fuzzy Sets and Systems*, vol. 49, no. 2, pp. 237-242, 1992. https://doi.org/10.1016/0165-0114(92)90329-3

[4] M. S. El Naschie, "Advanced prerequisite for E-infinity theory," *Chaos, Solitons & Fractals*, vol. 30, no. 3, pp. 636-641, 2006. https://doi.org/10.1016/j.chaos.2006.04.044

[5] A. A. Ramadan. ?Smooth topological spaces," *Fuzzy Sets and Systems*, vol. 48, no. 3, pp. 371-375, 1992. https://doi.org/10.1016/0165-0114(92)90352-5

[6] M. S. El Naschie, "A review of E-infinity theory and the mass spectrum of high energy particle physics," *Chaos, Solitons & Fractals*, vol. 19, no. 1, pp. 209-236, 2004. https://doi.org/10.1016/S0960-0779(03)00278-9

[7] M. S. El Naschie, "Elementary prerequisite for E-infinity (recommended background readings in nonlinear dynamics, geometry and topology)," *Chaos, Solitons & Fractals*, vol. 30, no. 3, pp. 579-606, 2006. https://doi.org/10.1016/j.chaos.2006.03.030

[8] M. S. El Naschie, "On a fuzzy Kahler-like manifold which is consistent with the two slit experiment," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 6, no. 2, pp. 95-98, 2005. https://doi.org/10.1515/IJNSNS.2005.6.2.95

[9] M. S. El Naschie, "On the uncertainty of Cantorian geometry and the two-slit experiment," *Chaos, Solitons & Fractals*, vol. 30, no. 3, pp. 517-529, 1998. https://doi.org/10.1016/S0960-0779(97)00150-1

[10] M. S. El Naschie, "On the unification of heterotic strings, M theory and E(∞) theory," *Chaos, Solitons & Fractals*, vol. 11, no. 14, pp. 2397-2408, 2000. https://doi.org/10.1016/S0960-0779(00)00108-9

[11] M. S. El Naschie, "Quantum gravity from descriptive set theory," *Chaos, Solitons & Fractals*, vol. 19, no. 5, pp. 1339-1344, 2004. https://doi.org/10.1016/j.chaos.2003.08.009

[12] M. S. El Naschie, "Quantum gravity, Clifford algebras, fuzzy set theory and the fundamental constants of nature," *Chaos, Solitons & Fractals*, vol. 20, no. 3, pp. 437-450, 2004. https://doi.org/10.1016/j.chaos.2003.09.029

[13] M. S. El Naschie, "The simplictic vacuum, exotic quasi-particles and gravitational instanton," *Chaos, Solitons & Fractals*, vol. 22, no. 1, pp. 1-11, 2004. https://doi.org/10.1016/j.chaos.2004.01.015

[14] M. S. El Naschie, "Topics in the mathematical physics of E-infinity theory," *Chaos, Solitons & Fractals*, vol. 30, no. 3, pp. 656-663, 2006. https://doi.org/10.1016/j.chaos.2006.04.043

[15] A. M. Zahran, S. A. Abd El-Baki, and Y. M. Saber, "Decomposition of fuzzy ideal continuity via fuzzy idealization," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 9, no. 2, pp. 83-93, 2009. https://doi.org/10.5391/IJFIS.2009.9.2.083

[16] A. M. Zahran, S. E. Abbas, S. A. Abd El-baki, and Y. M. Saber, "Decomposition of fuzzy continuity and fuzzy ideal continuity via fuzzy idealization," *Chaos, Solitons & Fractals*, vol. 42, no. 5, pp. 3064-3077, 2009. https://doi.org/10.1016/j.chaos.2009.04.010

[17] S. A. Abd El-Baki and Y. M. Saber, "Fuzzy extremally disconnected ideal topological spaces," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 10, no. 1, pp. 1-6, 2010.

[18] Y. C. Kim and J. M. Ko, "r-generalized fuzzy closed sets," *Journal of Fuzzy Mathematics*, vol. 12, no. 1, pp. 7-21, 2004.

[19] Y. M. Saber and M. A. Abdel-Sattar, "Ideals on fuzzy topological spaces," *Applied Mathematical Sciences*, vol. 8, no. 34, pp. 1667-1691, 2014. https://doi.org/10.12988/ams.2014.33194

[20] A. A. Ramadan, S. E. Abbas, and Y. C. Kim, "Fuzzy irresolute functions in smooth fuzzy topological space," *Journal of Fuzzy Mathematics*, vol. 9, no. 4, pp. 865-877, 2001.

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