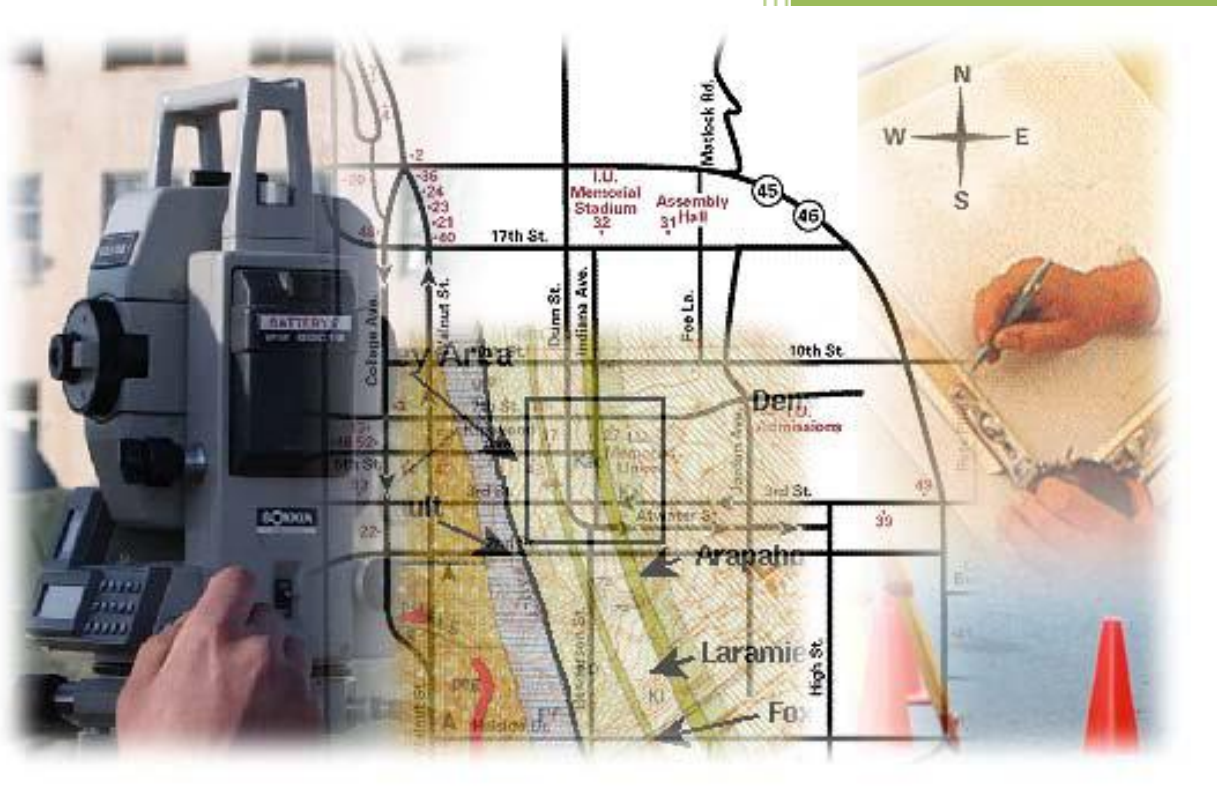


CE 370

Surveying I



Dr. SaMeH S. Ahmed

College of Engineering – MU 2018-19/2

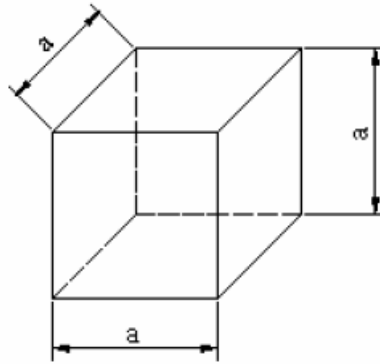
CE 370

Chapter 5

Volumes

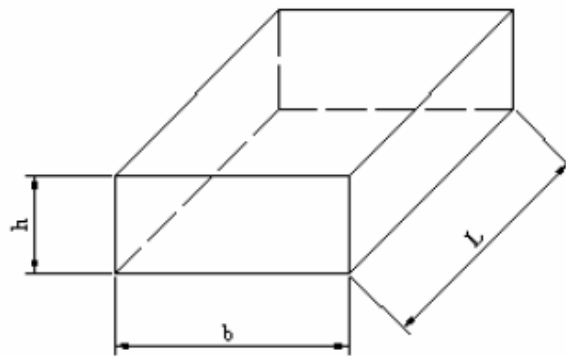
5.1 Volume of regular shapes

5.1 Cube



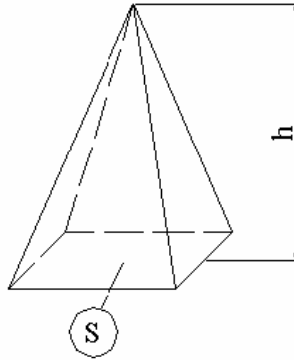
$$V = a^3$$

5.2 Cuboids



$$V = (L \cdot b \cdot h)$$

5.3 Pyramid



$$V = \frac{h}{3} (S)$$

S = area of the base

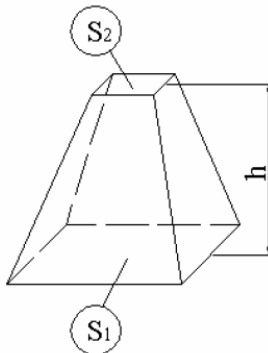
Example #1:

A square pyramid has a height of 9 meters. If a side of the base measures 4 meters, what is the volume of the pyramid?

Since the base is a square, area of the base = $4 \times 4 = 16 \text{ m}^2$

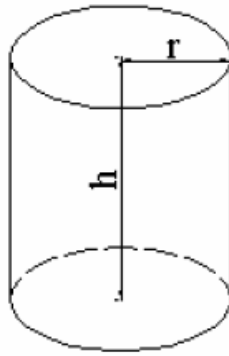
Volume of the pyramid = $(B \times h)/3 = (16 \times 9)/3 = 144/3 = 48 \text{ m}^3$

5.4 Harvested Pyramid



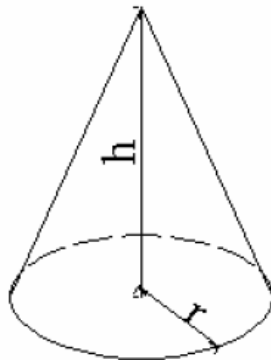
$$V = \frac{h}{3} (S_1 + S_2 + \sqrt{S_1 S_2})$$

5.5 Cylinder



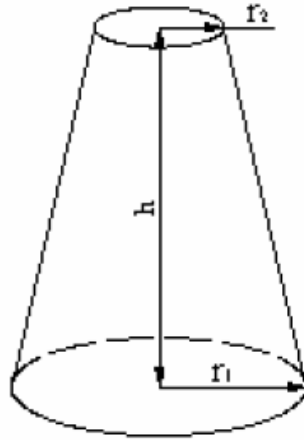
$$V = (\pi r^2 h)$$

5.6 Cone



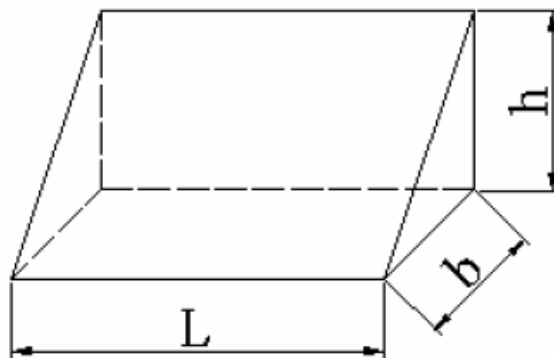
$$V = \frac{1}{3} (\pi r^2 h)$$

5.7 Harvested Cone



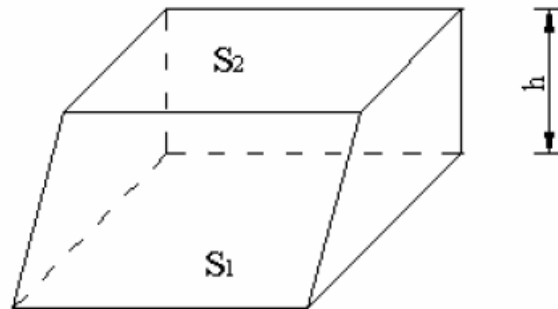
$$V = \frac{\pi}{3} (r_1^2 + r_2^2 + r_1 r_2) \cdot h$$

5.8 Prism



$$V = \frac{1}{2} (L \cdot b \cdot h)$$

5.9 Harvested Prism



$$V = \frac{h}{2}(S_1 + S_2)$$

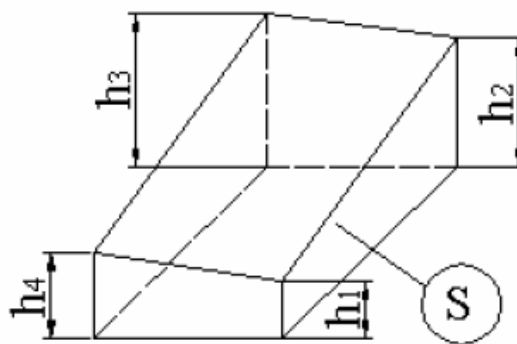
If (h) is quit high (the difference in elevation between the two surfaces, we apply the following equation)

$$V = \frac{h}{6}(S_1 + S_2 + 4S)$$

Where, S = average area between S_1 and S_2

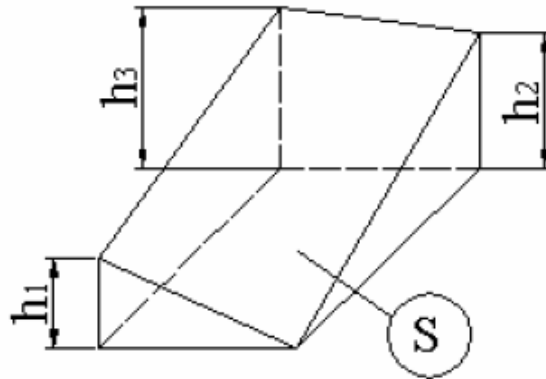
$$S = \left(\frac{l_1+b_1}{2}\right) \cdot \left(\frac{l_2+b_2}{2}\right)$$

5.10 Harvested Four Parallel Rectangles



$$V = S \left(\frac{h_1+h_2+h_3+h_4}{4} \right)$$

5.11 Harvested Three Parallel Rectangles



$$V = S \left(\frac{h_1 + h_2 + h_3}{3} \right)$$

Examples

1- The elevation of a flat land is 40 m; It has a hole at a level of 22 m. If the base of the hole is rectangle of 20 x 10 m and the land surface is 30 x 15 m. What is the volume of the excavated soil from that hole?

Solution:

$$S_1 = \text{area of the rectangle at the land level} = 15 \times 30 = 450 \text{ m}^2$$

$$S_2 = \text{area of the rectangle at the hole base level} = 20 \times 10 = 200 \text{ m}^2$$

$$h = \text{height} = 40 - 22 = 18 \text{ m}$$

V = volume of the excavated soil = ?

$$V = (S_1 + S_2) \frac{h}{2}$$

$$V = (450 + 200) \frac{18}{2} = 5850 \text{ m}^3$$

Another solution:

Using the prism method where:

$$\text{Average bases area} = S = \left(\frac{l_1+b_1}{2}\right) \cdot \left(\frac{l_2+b_2}{2}\right)$$

$$S = \left(\frac{30+20}{2}\right) \cdot \left(\frac{15+10}{2}\right) = 25 \times 12.5 = 312.5 \text{ m}^2$$

Since,

$$V = \frac{h}{6} (S_1 + S_2 + 4S)$$

$$V = \frac{18}{6} (450 + 200 + 4 \times 312.5) = \underline{5700 \text{ m}^3}$$

Note: the difference in the two calculated values is 150 m^3 which equivalent to 2.6% . This difference is decreased as the two surfaces are getting close to each other.

2- A small pile is formed from a brought out soil that taken from a hole, if the base of the formed pile is trapezoid and the lengths of its two bases are 32 and 24 m. and its height is 9 m. The upper surface is also in the form of trapezoid. The lengths of the two bases of the upper trapezoid are 12 and 8 m. and its height is 5 m. If the height of the pile is 6 m. find the volume of the brought soil?

Solution

$$S_1 = \left(\frac{32 + 24}{2}\right) \times 9 = 252 \text{ m}^2$$

$$S_2 = \left(\frac{12 + 8}{2}\right) \times 5 = 50 \text{ m}^2$$

$$V = (S_1 + S_2) \frac{h}{2}$$

$$V = (252 + 50) \frac{6}{2} = 906 \text{ m}^3$$

Another solution:

Using the prism method where:

$$\text{Average bases area (trapezoid shapes)} = S = \left[\left(\frac{32+12}{2} \right) + \left(\frac{24+8}{2} \right) \right] \times \left(\frac{9+5}{2} \right) \frac{1}{2} = 133 \text{ m}^2$$

Since,

$$V = \frac{h}{6} (S_1 + S_2 + 4S)$$

$$V = \frac{6}{6} (252 + 50 + 4 \times 133) = \underline{834 \text{ m}^3}$$

Despite the difference between the two volumes calculated in this example is less than the difference in example (1), but the percentage is higher due to the big difference in the surface areas.

تذكر : مساحة شبة المنحرف = نصف مجموع الضلعين المتوازيين في الارتفاع