Majmaah University **College of Engineering**

Civil and Environmental Engineering Department **2018-19/2**

Sheet (4)

Chain and Tape Corrections

Study these examples very well; you will have a quiz next week

1. Correction of Absolute Length

Let,

l = designated length of the tape

l_a = absolute length of the tape

Then correction per chain length

$$c = l_a - l$$

Hence, if the total length measured is L, the correction is

$$C_a = L \cdot \frac{c}{l}$$

If absolute of tape "l_a" is greater, correction is +ve., and if negative the correction is also negative. Thus correct length L^* is given by:

$$\boldsymbol{L^* = L + C_a = L \cdot \left(1 + \frac{c}{l}\right)}$$

If *A* is the measured area with incorrect tape, the correct area is given by

$$\boldsymbol{A^* = A \left(\frac{L^*}{L}\right)^2}$$

2. Correction of Slope

If length measured *"L"* and the difference in the levels of first and last point *"h"* are given then correction for slope is:

$$C_{sl} = L - \sqrt{L^2 - h^2}$$

Approximate formula for slope..

$$\text{Correction} = \frac{h^2}{2L}$$

If measured length *"L"* and its slop θ and are given, then

$$\boldsymbol{C_{sl} = L - L\cos\theta}$$

This correction is always subtractive.

3. Correction for Temperature

Let

α = Coefficient of thermal expansion of the material of tape
T_m = Mean temperature during measurement
T_0 = Temperature at which tape is standardized, and
L = Measured length

Then, temperature correction C_t , is given by

$$\boldsymbol{C_t = L\,\alpha\,(T_m - T_0)}$$

It is positive, if $T_m > T_0$ and is negative if $T_m < T_0$

4. Correction for Pull

Let,

E – Young's modulus of the material of tape
A – Cross-sectional area of the tape
P – Pull applied during measurement
P_0 - Standard pull, and
L - Measured length of chain

Then, the correction for pull C_p is given by

$$C_p = \frac{\boldsymbol{(P - P_0)L}}{\boldsymbol{AE}}$$

It is positive, if $\boldsymbol{P} > \boldsymbol{P_0}$ and is negative if $\boldsymbol{P} < \boldsymbol{P_0}$

5. Correction for Sag

While taking reading, if the tape is suspended between two supports, the tape sags under its own weight. Hence, measured length is more than the actual length. Hence, *this correction is subtractive*. This correction is given by:

$$C_s = \frac{\boldsymbol{1}}{\boldsymbol{24}}\left(\frac{\boldsymbol{W}}{\boldsymbol{P}}\right)^{\boldsymbol{2}} \boldsymbol{L}$$

Where:

W – The weight of the tape per span length

P – The pull applied during the measurement

L – Measured length

Examples

Example 1:

A distance of 2000 m was measured by 30 m tape. After the measurement, the tape was found to be 10 cm longer. It was found to be 15 cm longer after another 500 m was measured. If the length of the tape was correct before the measurement, determine the exact length of the whole measurement.

Solution

For the first 2000 m length:

Average correction per tape length

$= \frac{0+10}{2}$ = 5 cm = 0.05 m

Correction for measured length

$$C_a = L \cdot \frac{c}{l}$$

$$C_a = 2000\, x \frac{0.05}{30} = 3.33\, m$$

True length = 2000 + 3.33 = 2003.33 m

For the next 500 m length:

Average correction per tape length

= $\frac{10+15}{2}$ = 12.5 cm = 0.125 m

Correction for measured length

$$C_a = 500\, x \frac{0.125}{30} = 2.08\, m$$

True length = 500 + 2.08 = 503.08 m

Exact length of the whole line = 2003.33 + 502.08 = 22505.41 m

Example 2:

The length of a survey line when measured with a chain of nominal 20 m length was found to be 841.5 m. When the chain was compared with a standard it was found to be 0.1 too long. Compute the correct length of the line.

Solution

Correction for chain length = 0.1 m

Measured length L = 841.5 m

Nominal length of chain = 20 m

$$C_a = 841.5\, x \frac{0.1}{20} = 4.21\, m$$

Actual length of line = 841.5 + 4.21 = ***845.71 m***

Example 3:

A tape was tested before starting the survey and was found to be exactly 20 m. At the end of the survey it was tested again and was found to be 20.12 m. Area of the plan of the field drawn to a scale 1 cm = 6 m was 50.4 cm^2. Find the true area of the field in square meters.

Solution

Initial length of the tape = 20 m

Length at the end of survey = 20.12 m

∴ Final correction for tape length = 20.12 – 20.0 = 0.12 m

Average correction for tape length = $\frac{0+0.12}{2}$ = 0.06 m

∴ True length of tape $\boldsymbol{L^* = L + C_a}$ = 20 + 0.06 = 20.06 m

Measured area on plan = 50.4 cm^2

Scale = 1 cm = 6 m

Measured area on ground = 50.4 x 6^2 = 1814.4 m^2

True area on the ground

$$\boldsymbol{A^* = A\left(\frac{L^*}{L}\right)^2}$$

$$\boldsymbol{A^* = 1814.4\left(\frac{20.06}{20}\right)^2} = \underline{\textbf{1825.3 m}^2}$$

Example 4:

An old map was drawn to a scale of 1:1000. The field was surveyed with a 30 m tape which was actually 5 cm too long. An original length of 10 cm has now shrunk to 9.75 cm on the plan. The plan area measured with a planimeter is 97.03 cm^2. Calculate the actual area of land in hectares.

Solution

Original length on plan = 10 cm

Shrunk length = 9.75 cm

Measured area on shrunk plan = 97.03 cm^2

10 9.75

$(10)^2$ $(9.75)^2$

X 97.03

∴ Original area measured on plan = $\mathbf{97.03\ \left(\frac{10}{9.75}\right)^2}$ **= 102.07 cm^2**

Scale 1: 1000

∴ Measured area on ground A = $(10)^2$ x 102.07 = 10207.0 m^2

Nominal length of tape = 30 m

Actual length of tape = 30 + 0.05 = 30.05 m

Actual area A* = $\mathbf{10207\ \left(\frac{30.05}{30}\right)^2}$

= 10241 m^2

= **1.0241 hectare**

Example 5:

The following slope distances were measured with a 50 m tape.

Slope Distance	Difference in Elevation of End Point
46.2 m	3.2 m
38.5 m	4.3 m
42.6 m	5.4m

Find the total horizontal distance measured using (a) exact formula (b) approximate formula.

Solution

(a) Exact formula

$$L = \sqrt{46.2^2 - 3.2^2} + \sqrt{38.5^2 - 4.3^2} + \sqrt{42.6^2 - 5.4^2} = 126.605 \text{ m}$$

(b) Approximate formula

$$\text{Correction} = \frac{h_1^2}{2l_1} + \frac{h_2^2}{2l_2} + \frac{h_3^2}{2l_3}$$

$$= \frac{3.2^2}{2\ x\ 46.2} + \frac{4.3^2}{2\ x\ 38.5} + \frac{5.4^2}{2\ x\ 42.6}$$

= 0.693

Total length measured = 46.2 + 38.5 + 42.6

= 127.3 m

Total horizontal length = 127.3 – 0.693

= **126.607 m**

Example 6:

The length of line measured on a slope of 14^0 was recorded as 272 m. It was subsequently found that the chain was 20.2 m long instead of 20 m. calculate the correct horizontal length.

Solution

Correction required per chain length = 0.2 m

∴ Correction to Slope Length Measured

$$= 272 \times \frac{0.2}{20} = 2.72 \text{ m}$$

Hence, correct slope length = 272 + 2.72 = 274.72 m

Slope is at $\theta = 14^o$

∴ *Correct horizontal length*

$$= 274.72 \cos 14^o$$

$$= \mathbf{266.56 \text{ m}}$$

Example 7:

A tape 20 m long of standard length at 84°F was used to measure a line, the mean temperature during the measurement being 63°F. The measured distance was 1021.40 meters, the following being the slopes:

2° 15′	for	150 m
4° 18′	for	175 m
1° 20′	for	100 m
7° 18′	for	250 m
3° 10′	for	300 m
5° 00′	for	146.4 m

Find the true length of the line if the coefficient of thermal expansion is 62×10^{-7} per 1°F.

Solution

Measured horizontal distance =
= 100 cos 2° 15′ + 125 cos 4° 18′ + 50 cos 1° 20′ + 200 cos 7° 18′ + 250 cos 3° 10′ + 96.4 cos 5° 00′

= 818.588 m

Temperature Correction =

$$C_t = L\,\alpha\,(T_m - T_0)$$

= 818.588 x 62 x 10^{-7} (63 – 84)
= -0.107

Corrected Horizontal length = 818.588 -0.107
= **818.48 m**

Example 8:

To measure a base line, a steel tape 30 m long standardized at 15^{o}C with a pull of 100 N was used Find the correction per tape length if the temperature at the time of measurement was 20^{o}C and the pull exerted was 160 N. if the length of 250 m is measured on a slope of 1 in 4, find the horizontal length. Take E = 2.1 x 10^{5} N/mm^{2}
α = 11.2 x 10^{-6} /C^{o} and cross sectional area of tape = 0.08 cm^{2}

Solution

Length of tape = 30 m

α = 11.2 x 10^{-6} /C^{o}

T_m = 20^{o}C and T_0 = 15^{o}C

Correction for temperature

$$= C_t = L\,\alpha\,(T_m - T_0)$$

= 30 x 11.2 x 10^{-6} (20 – 15)

= 1680 x 10^{-6} = **1.680 x 10^{-3} m**

Correction for pull = $C_p = \frac{(P - P_0)L}{AE}$

P = 160 N

P_0 = 100 N

A = 0.08 cm^{2}

E = 2.1 x 10^{5} N/mm^{2} = 2.1 x 10^{7} N/cm^{2}

Correction for pull = $C_p = \frac{(160 - 100)30}{0.08 \text{ x } 2.1 \text{ x } 10^{7}}$ **= 1.071 x 10^{-3} m**

Combined correction for temperature and pull

= 1.680 x 10^{-3} + 1.071 x 10^{-3} **= 2.751 x 10^{-3}** m per chain length

Slope Length Measured = 250 m

Correction for Sloping length = 250 x $\frac{2.751 \text{ x } 10^{-3}}{30}$ = 0.023 m

Correct sloping length = 250 + 0.023 = 250.023 m

The slope is 1:4 i.e., θ = tan^{-1} ¼ = 14.036^{0}

Correct horizontal length = 250.023 cos 14.036^{0} = **142.56 m**

Example 9:

Calculate the sag correction for a 30 m steel tape under a pull of 100 N in three equal spans of 10 m each. Unit weight of steel is 786 kN/m^3. Area of cross-section of tape is 8 mm^2.

Solution

W = wt of the tape per span length

= 78.6 x 10 x (8 x 10^{-6})

= 6288 x 10^{-6} kN = 6.288 N

(Note: 1 mm^2 = $(0.001)^2$ m^2 = 1 x 10^{-6} m^2)

P = 100 N

L = 10000 mm for each span

Correction for sag for each span

$$C_s = \frac{1}{24}\left(\frac{W}{P}\right)^2 L$$

=

$$C_s = \frac{1}{24}\left(\frac{6.288}{100}\right)^2 10000$$

= 1.6475 mm

Correction for sag for the three spans

= 3 x 1.6475

= 4.94 mm

Example 10:

A 30 m steel tape was standardized under 40 N pull at 66^0F. It was suspended in 5 equal spans during measurement. The temperature during the measurement was 92^0F and the pull exerted was 100 N. the area of cross-section of the tape was 8 mm^2. The unit weight of the steel tape is 78 kN/m^3. Take coefficient of thermal expansion $\alpha = 6.3 \times 10^{-6}$ /F^o and Young's modulus E =2×10^{-5} N/mm^2. Find the true length of the tape.

Solution

Correction for temperature

$$= C_t = L\,\alpha\,(T_m - T_0)$$

$$= 30 \times 6.3 \times 10^{-6} \,(\, 92 - 66\,)$$

$$= 4914 \times 10^{-6} \text{ m} = \textbf{4.914 mm (+ve)}$$

Correction for pull = $C_p = \frac{(P - P_0)L}{AE}$

$$C_p = \frac{(100 - 40)30\,x\,1000}{8 \text{ x } 2 \text{ x } 10^5} = \textbf{1.125 mm (+ve)}$$

Correction for sag:

Unit weight = 78 kN/m^3 = 78 x 1000 x 10^{-9} N/mm^3

= 78 x 10^{-6} N/mm^3

Each span is of 30/5 = 6 m = 6000 mm

Weight of tape per span of 6 m = 78 x 10^{-6} x 8 x 6000 = 3.744 N

Correction for sag in mm unit per span

$$C_s = \frac{1}{24}\left(\frac{3.744}{100}\right)^2 6000 = 0.3504 \text{ mm}$$

Since there are five spans,

Total C_s = 5 x 0.350 = 1.752 mm (-ve)

So, Total correction = 4.194 + 1.125 – 1.752 = 4.287 mm= 0.00429 m

True length of tape = 30 + 0.00429 = **30.00429 m**